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GRANT NAG-3340  
JULY 1983



25th Anniversary  
1958-1983

NASA



## NASA Contractor Report 3705

# Scatter of Elastic Waves by a Thin Flat Elliptical Inhomogeneity

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Prepared for  
Lewis Research Center  
under Grant NAG-3340



National Aeronautics  
and Space Administration

**Scientific and Technical  
Information Branch**

1983

## INTRODUCTION

The interaction of elastic waves with material inhomogeneities such as second-phase particles and cracks plays an important role in material and fracture characterization by quantitative ultrasonics [1,2]. These inhomogeneities represent mis-matches in material moduli as well as mass density compared with the surrounding matrix material and thereby generate elastodynamic fields in addition to the fields when they are absent. These fields are referred to as the scattered fields when measured far from the inhomogeneities. When the non-dimensional wavenumber is greater than unity,  $ka > 1$ , the term diffraction is usually used in place of scattering.

Diffraction of elastic waves by thin flat objects, cracks in particular, has been an actively pursued subject for sometime by theoretical mechanicians. Due to the complexity of the presence of a third dimension, most work is confined to essentially a two-dimensional situation. Recently, Teitel [3], Gubernatis and Domany [4], and Datta [5], using the quasi-static approach and asymptotic expansion approach, respectively, obtained results for an elliptical crack that are valid in the Rayleigh limit,  $ka \ll 1$ . Budiansky and Rice [6] gave a general integral formulation for the dynamic response of an isolated three-dimensional crack. Employing the geometric diffraction theory [7], Gautesan, Achenbach and McMaken [8] investigated the diffraction by elliptical cracks valid for the region of short wavelength,  $ka \gg 1$ . A summary of the comparison with experimental results is given in [9] for a penny-shaped crack and [10] for an elliptical crack. For the region of medium wavelength, results available are obtained mostly by numerical methods through a matrix approach.

In the present paper the scattering of elastic waves by an isolated, flat, thin, and elliptical inhomogeneity is studied by employing the extended version of the method of equivalent inclusion. The method of equivalent inclusion was originally developed by Eshelby [11,12] for the determination of elastostatic fields inside and outside an ellipsoidal inhomogeneity. Wheeler and Mura [13] first extended this method to study the elastodynamic response of composites where the mis-match in mass density is ignored. Recently, Fu and Mura [14] gave a complete formulation to extend the method of equivalent inclusion for elastodynamic problems. They presented the equivalence conditions that allow the elastodynamic fields inside and outside an inhomogeneity to be determined. In addition to the presence of eigenstrains,  $\epsilon_{ij}^*(1)$ , the concept of "eigenforces"  $\pi_j^{*1}$  is also introduced.

The solution procedure given in [14] requires the evaluation of certain volume integrals that are associated with the inhomogeneous Helmholtz equation. A method for evaluating these integrals is given by Fu and Mura [15]. It is easily seen from Ref. [15] that the results in Ref. [14] reduce to the elastostatic solution when the frequency approaches zero,  $\omega \rightarrow 0$ . It is further noted that since all the geometric information are contained in these volume integrals, scattering of inhomogeneity of any geometry can be obtained simply by evaluating these integrals.

The case of the scattering of a perfect sphere is studied in detail by Sheu and Fu [16] and compared with classical results of Ying and Truell [17]. It was shown that the comparison is good up to  $ka$  about two when uniform distribution of eigenstrains and eigenforces are assumed.

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<sup>1</sup>Although this term was not used in Ref. [14], it appears to be appropriate for this quantity as it can be seen from its definition  $\pi_j^* = C_{jkr s} \epsilon_{rs, k}^{*(2)} = \Delta \rho \ddot{u}_j$ . Its unit is of course the same as body force, i.e. force per unit volume.

In this paper the solution for a flat, thin elliptical inhomogeneity is obtained by collapsing the ellipsoid, say letting  $a_3 \rightarrow 0$ . Since the eigenstrains and eigenforces become infinite the limiting concept described by Mura [18] is employed.

## ELASTODYNAMIC FIELDS OF AN ISOLATED INHOMOGENEITY

For convenience in reading, it appears to be worthwhile to summarize the solution given in Fu and Mura [14]. Consider the physical problem of an isolated inhomogeneity embedded in an infinite elastic solid which is subjected to a plane time-harmonic incident wave field as depicted in Fig. 1. Replacing the inhomogeneity with the same material as that of the surrounding medium, with moduli  $C_{jkr s}$  and mass density  $\rho$ , and include in this region a distribution of eigenstrains and eigenforces, the physical problem is now replaced by the equivalent inclusion problem.

Following Fu and Mura [14] the total field is now obtained as the superposition of the incident field and the field induced by the presence of the mis-matches in moduli and in mass density written in terms of eigenstrains  $\epsilon_{ij}^{*(1)}$  and eigenforces,  $\pi_j^*$ :

$$\tilde{F} = \tilde{F}^{(i)} + \tilde{F}^{(m)} \quad (1)$$

where  $\tilde{F}$  denotes either the displacement field  $u_j$ , the strain field  $\epsilon_{ij}$ , or the stress field  $\sigma_{ij}$ . The superscripts (i) and (m) denote "incident" and "mis-match", respectively.

For uniform distributions of eigenstrains and eigenforces, the fields can be obtained as:

$$u_m^{(m)}(\bar{r}) = -\pi_j^* S_{jm}(\bar{r}) - C_{jkr s} \epsilon_{rs}^{*(1)} S_{jm,k}(\bar{r}) \quad (2)$$

$$\epsilon_{mn}^{(m)} = (u_{m,n}^{(m)} + u_{n,m}^{(m)})/2 \quad (3)$$

$$\sigma_{pq}^{(m)} = C_{pqmn} \epsilon_{mn}^{(m)} \quad (4)$$

where a comma denotes partial differentiation and

$$S_{jm}(\vec{r}) = \int_{\Omega} g_{jm}(\vec{r}-\vec{r}')dV' \quad (5)$$

in which  $g_{jm}$  is the spatial part of the free space Green's function and  $\Omega$  is the region occupied by the inhomogeneity. It is noted that the integrals  $S_{jm}$  and their derivatives must be evaluated for the region  $\vec{r} > \vec{r}'$  and for the region  $\vec{r} < \vec{r}'$ , Ref. [15]. The solution form represented in Eqs. (1-5) gives the fields inside and outside an isolated inhomogeneity of arbitrary shape.

# AN ISOLATED FLAT THIN INHOMOGENEITY

Let the incident displacement field be longitudinal and of frequency  $\omega$ , amplitude  $u_0$ :

$$u_j^{(i)} = u_0 q_j \exp[i\alpha x_i q_i - i\omega t] \quad (6)$$

where  $i^2 = -1$  and  $q_j$  is the unit vector in the normal direction of the plane time harmonic wave. For a linear isotropic medium, the spatial part of the free space Green's function is well known. Substituting  $g_{jm}(\bar{r}-\bar{r}')$  in Eq. (2) and using the limiting concept<sup>1</sup>

$$\lim_{a_3 \rightarrow 0} a_3 \pi_j^* = A_j \quad \text{constants} \quad (7)$$

$$\lim_{a_3 \rightarrow 0} a_3 \epsilon_{ij}^{*(1)} = B_{ij} \quad \text{constants} \quad (8)$$

the scattered displacement  $u_m^{(s)}(\bar{r}, t)$  from a thin elliptical flat inhomogeneity can easily be obtained as:

$$\begin{aligned} \frac{u_m^{(s)}(\bar{r}, t)}{(\alpha a_1)^3 u_0} &= \left[ \frac{u_m^{(m)}(\bar{r}, t)}{(\alpha a_1)^3 u_0} \right]_{r \rightarrow \infty} \\ &= [(C G_m \exp i \alpha r) / \alpha r + (D H_m \exp i \beta r) / \beta r] \exp(-i\omega t) \end{aligned} \quad (9)$$

where

$$G_m = - (a_2/a_1) [\ell_m \ell_j A_j^* (\Delta \rho / \rho) + (1 - 2\alpha^2/\beta^2) \ell_m B_{jj}^* + 2(\alpha^2/\beta^2) \ell_m \ell_k \ell_j B_{kj}^*] \quad (10)$$

$$\begin{aligned} H_m &= (a_2/a_1) [(\beta/\alpha)^3 (\ell_m \ell_j - \delta_{mj}) A_j^* (\Delta \rho / \rho) \\ &\quad - 2(\beta/\alpha)^2 \ell_k B_{km}^* + 2(\beta/\alpha)^2 \ell_m \ell_k \ell_j B_{kj}^*] \end{aligned} \quad (11)$$

<sup>1</sup>This concept is discussed in [18] in the light of Eq. (8).

$$C = \sum_{n=0,2}^{\infty} \sum_{\ell=0,2}^n \frac{(-1)^n (\alpha a_1)^n (a_2/a_1)^{n-\ell} \ell_1^{\ell} \ell_2^{n-\ell} (n/2)!}{(n+3)(n+1)n! (\ell/2)! (n-\ell)/2!} \quad (12)$$

$$D = \sum_{n=0,2}^{\infty} \sum_{\ell=0,2}^n \frac{(-1)^n (\beta a_1)^n (a_2/a_1)^{n-\ell} \ell_1^{\ell} \ell_2^{n-\ell} (n/2)!}{(n+3)(n+1)n! (\ell/2)! (n-\ell)/2!} \quad (13)$$

in which  $m, j, k=1,2,3$ , and  $(a_1, a_2)$ ,  $\ell_m$ ,  $\alpha$ ,  $\beta$ , and  $\Delta \rho$  denote the semi-axes of elliptical inhomogeneity, direction cosines of scattered displacements, longitudinal wavenumber, shear wavenumber, and mass density mis-match  $(\rho' - \rho)$ , respectively. Also,  $A_j^*$  and  $B_{jk}^*$  are the reduced non-dimensional form of  $A_j$  and  $B_{jk}$ , respectively, defined as follows:

$$A_j^* = A_j / (\Delta \rho \omega^2 u_o) \quad (14)$$

$$B_{jk}^* = -B_{jk} / (i \alpha u_o) \quad (15)$$

Expressions for the differential cross section  $dP(\omega)/d\Omega$  and total cross section  $P(\omega)$  can be obtained as before

$$\frac{dP(\omega)}{d\Omega} = \sigma^P(\theta, \phi) + (\alpha/\beta) \sigma^S(\theta, \phi) \quad (16)$$

$$P(\omega) = \int_{\Sigma} \sigma^P(\theta, \phi) + (\alpha/\beta) \sigma^S(\theta, \phi) d\Omega \quad (17)$$

where  $d\Omega$  is the differential element of solid angle and

$$\alpha^2 \sigma^P(\theta, \phi) = (\alpha a_1)^6 [C \ G_m] [\overline{C \ G_m}] \quad (18)$$

$$\beta^2 \sigma^S(\theta, \phi) = (\alpha a_1)^6 [D \ H_m] [\overline{D \ H_m}] \quad (19)$$

in which the super bars denote complex conjugate. It is noted that the constants  $A_j^*$  and  $B_{jk}^*$  must be evaluated from the equivalence conditions given in Ref. [14] with the use of the limiting concepts in Eqs. (7,8) and of the integration method developed in Ref. [15].

## DETERMINATION OF $A_j^*$ AND $B_{jk}^*$

In Eqs. (9-13) the scattered displacement field is given in terms of the "reduced" form of the eigenforces and eigenstrains, i.e.  $A_j^*$  and  $B_{jk}^*$ , see Eqs. (7,8,14,15). These constants must in turn be determined from the equivalence conditions, Eq. (14), Ref. [14]. Writing the incident wave field in a Taylor series the governing simultaneous algebraic equations can be easily obtained. Since  $f_{mij}[0]$  and  $F_{mij}[0]$  vanish automatically, these governing equations become uncoupled and lead to a three by three system for  $A_j^*$  and a six by six system for  $B_{jk}^*$ , where Eqs. (25,26) in Ref. [14] are used. For a linear elastic medium, they are:

$$\Delta\rho\omega^2 u_{ofjs}[0]A_j^* + A_s^* = -q_s \quad (20)$$

$$\begin{aligned} & \{\Delta\lambda\delta_{st}D_{mmjk}[0] + 2\Delta\mu D_{stjk}[0]\}B_{jk}^* + \{\lambda\delta_{st}B_{mm}^* + 2\mu B_{st}^*\} \\ & = -\{\Delta\lambda\delta_{st}q_m q_m + 2\Delta\mu q_s q_t\} \end{aligned} \quad (21)$$

where the subscripts  $s, t, m, j, k = 1, 2, 3$ , repeated subscripts denote sum from 1 to 3, and

$$4\pi\rho\omega^2 f_{js}(\vec{r}) = -\beta^2\phi_{js} + \psi_{,mj} - \phi_{,mj} \quad (22)$$

$$\begin{aligned} 4\pi\rho\omega^2 D_{stjk}(\vec{r}) &= 2\mu[\psi_{,stjk} + \phi_{,stjk}] \\ &\quad - \mu\beta^2[\phi_{,jt}\delta_{ks} + \phi_{,js}\delta_{kt}] \\ &\quad - \lambda\alpha^2\psi_{,mn}\delta_{jk} \end{aligned} \quad (23)$$

$$\Delta\lambda = \lambda' - \lambda, \quad \Delta\mu = \mu' - \mu, \quad \Delta\rho = \rho' - \rho$$

The  $\phi$ - and  $\psi$ -integrals and their derivatives are evaluated by the method suggested in Ref. [15]. Retaining terms up to  $(\alpha a_1)$  or  $(\beta a_1)$  of the fourth order, the constants are obtained as:

$$A_j^* = -q_j / \{u_0 \omega^2 \Delta \rho f_j[0] + 1\}, \text{ no sum on } j \quad (24)$$

$$f_j[0] = (f_{11}[0], f_{22}[0], f_{33}[0])$$

$$\{B_i^*\} = [b_{ij}]^{-1} [c_j] \quad i, j=1, 2, 3 \quad (25)$$

$$\{B_{ij}^*\}_{i \neq j} = -q_i q_j / [1 + \xi(\phi_i[0] + \phi_j[0])], \text{ no sum on } i, j. \quad (26)$$

$$i, j=1, 2, 3 \quad \xi = 1/4\pi$$

$$\text{where in Eq. (25) } B_1^* = B_{11}^*, B_2^* = B_{22}^*, B_3^* = B_{33}^*, \quad (27)$$

$$c_j = -(\Delta\lambda + 2\Delta\mu) [1 + q_j^2], \quad j=1, 2, 3,$$

$$b_{11} = (\lambda + 2\mu) + \lambda\zeta\psi_{,jj}[0] + 2(\lambda + 2\mu)\xi\phi_{,11}[0] + 2\mu\zeta\psi_{,11}[0]$$

$$b_{12} = \lambda + \lambda\zeta\psi_{,jj}[0] + 2\mu\zeta\psi_{,11} + 2\xi\lambda\phi_{,22}[0]$$

$$b_{13} = \lambda + \lambda\zeta\psi_{,jj}[0] + 2\mu\zeta\psi_{,11}[0] + 2\xi\lambda\phi_{,33}[0]$$

$$b_{21} = \lambda + \lambda\zeta\psi_{,jj}[0] + 2\mu\zeta\psi_{,22}[0] + 2\xi\lambda\phi_{,11}[0]$$

$$b_{22} = (\lambda + 2\mu) + \lambda\zeta\psi_{,jj}[0] + 2\mu\zeta\psi_{,22}[0] + 2\xi(\lambda + 2\mu)\phi_{,22}[0]$$

$$b_{23} = \lambda + \lambda\zeta\psi_{,jj}[0] + 2\mu\zeta\psi_{,22}[0] + 2\xi\lambda\phi_{,33}[0]$$

$$b_{31} = \lambda + \lambda\zeta\psi_{,jj}[0] + 2\mu\zeta\psi_{,33}[0] + 2\xi\lambda\phi_{,11}[0]$$

$$b_{32} = \lambda + \lambda\zeta\psi_{,jj}[0] + 2\mu\zeta\psi_{,33}[0] + 2\xi\lambda\phi_{,22}[0]$$

$$b_{33} = (\lambda + 2\mu) + \lambda\zeta\psi_{,jj}[0] + 2\mu\zeta\psi_{,33}[0] + 2(\lambda + 2\mu)\xi\phi_{,33}[0]$$

$$\xi = 1/4\pi \quad \zeta = (1 - 2\alpha^2/\beta^2)/4\pi$$

Note that  $b_{ij} \neq b_{ji}$ . In Eq. (26),  $\phi_1 = \phi_{11}^{[0]}$ .  $\phi_2 = \phi_{22}[0]$ ,  $\phi_3 = \phi_{33}[0]$ .

The f-,  $\phi$ - and  $\psi$ - functions are given as:

$$4\pi\rho\omega^2 f_{js}[0] = -\beta^2\phi[0]\delta_{js} + \psi_{,js}[0] - \phi_{,js}[0] \quad (28)$$

$$\beta^2\phi[0] = \pi a_1 a_2 \beta^2 \{I_0 - ((\beta a_1)^2/16)I_1 + i(4/3)\beta(a_1^2 + a_2^2)\} \quad (29)$$

$$\phi_{,11}[0] = -[(\pi a_1^3 a_2 \beta^4)/12] \cdot I_1 \quad (30)$$

$$\phi_{,22}[0] = -[(\pi a_1 a_2^3 \beta^4)/12] \cdot I_2 \quad (31)$$

$$\phi_{,33}[0] = 0 \quad (32)$$

in which

$$I = \int_0^\infty \frac{d\psi}{\Delta(\psi)} = F(\theta, k) \quad (33)$$

$$I_1 = \int_0^\infty \frac{\psi d\psi}{(a_1^2 + \psi)\Delta(\psi)} = \frac{2}{a_1} \left\{ \frac{E(\theta, k)}{k^2} - \frac{k'^2}{k^2} F(\theta, k) \right\} \quad (34)$$

$$I_2 = \int_0^\infty \frac{\psi d\psi}{(a_2^2 + \psi)\Delta(\psi)} = \frac{2}{a_1} \left\{ \frac{F(\theta, k)}{k^2} - \frac{E(\theta, k)}{k^2} \right\} \quad (35)$$

$$F = \int_0^\theta \frac{d\omega}{(1-k^2 \sin^2 \omega)^{1/2}}, \quad E = \int_0^\theta (1-k^2 \sin^2 \omega)^{1/2} d\omega$$

and as  $a_3 \rightarrow 0$ ,  $\theta \rightarrow \pi/2$ ,  $k^2 \rightarrow (1-a_2^2/a_1^2)$  and  $k'^2 = (1-k^2) \rightarrow a_2^2/a_1^2$ , if  $a_1 > a_2$ . If  $a_3 \rightarrow 0$  and  $a_1 = a_2$ , we have  $I = \pi$ ,  $I_1 = I_2 = \pi/2a_1$ .

# EXAMPLES: SCATTERING OF A CRACK AT LONGWAVE LIMIT

Consider now the scattering of an elliptical crack as an example.  
Since the crack is a void,

$$\Delta\rho = -\rho, \Delta\lambda = -\lambda, \Delta\mu = -\mu$$

The constants  $A_j^*$  and  $B_{jk}^*$  can be easily obtained by solving the simultaneous equations, Eqs. (24-26):

$$A_j^* = -q_j(1-a_1a_2\beta^2F(\theta,k)/4) \quad (36)$$

$$B_{ij}^* = -q_iq_j[1+\xi(\phi_i + \phi_j)], \text{ if } i \neq j, \text{ no sum on } i,j. \quad (37)$$

$$B_{ij}^* = -(1+q_iq_j)[(\lambda+2\mu)(-\lambda+2\mu)]/2\mu(3\lambda+2\mu) \\ + \delta_{i3}\delta_{j3}(1+q_3q_3)[(\lambda+2\mu)(2\lambda)]/2\mu(3\lambda+2\mu) \quad \text{if } i=j, \text{ no sum on } i,j. \quad (38)$$

For ultrasonic applications, the longwave limit is always of interest, i.e. only terms up to the second order in  $(\alpha a)$  and  $(\beta a)$  are retained in the scattered displacement amplitudes. From Eqs. (9-13), it is observed that, this requires that only terms of  $(\alpha a)$  and  $(\beta a)$  to the zeroth order be retained in Eqs. (36-38). A direct substitution of Eqs. (36-38) into Eqs. (9-13) easily yields the expression for the scattered displacement amplitudes in the longwave limit. Simple manipulation yields:

$$CG_m = (a_2/3a_1)\ell_m\{-\ell_jq_j+(\alpha^2/\beta^2)(\ell_j\ell_kq_jq_k)\}_{j \neq k=1,2,3} \\ - (2-\beta^2/\alpha^2)(2-2\nu-\nu q_3^2)/(1+\nu)(1-2\nu) \\ + (1+\ell_k^2q_k^2)(1-3\nu)/(1+\nu) + (1+q_3^2)\ell_3^2(2\nu)/(1+\nu)\} \quad (39)$$

and

$$\begin{aligned}
DH_m = & (a_2/3a_1) \{ (\beta/\alpha)^3 (\ell_m \ell_j - \delta_{mj}) q_j \\
& + (\beta/\alpha)^2 [ -(\ell_m \ell_j \ell_k q_j q_k)_{j \neq k} + (2\ell_k q_k q_m)_{k \neq m} ] \\
& - (\beta/\alpha)^4 \ell_m [ (1 - (\ell_1 + \ell_2 + \ell_3) - \ell_j q_j^2 + \ell_k^2 q_k^2) (1 - 3\nu) / (1 + \nu) \\
& + (1 + q_3^2) (\ell_3^2 - \ell_3) (2\nu) / (1 + \nu) ] \}
\end{aligned} \tag{40}$$

where  $\nu$  is Poisson's ratio.

Let the incident wave be going from the negative  $z$ -axis to the positive  $z$ -axis, Fig. 2, it is clear that

$$q_1 = q_2 = 0 \quad q_3 = 1$$

For the forward and backward scattered displacements along the  $z$ -axis, the direction cosines are  $\ell_1 = \ell_2 = 0$  and  $\ell_3 = \pm 1$ , respectively.

The displacement amplitude can be found, for example, as follows:

$$u_1^{(s)} = u_2^{(s)} = 0 \tag{41a}$$

$$u_3^{(s)} / (\alpha a_1)^3 u_0 = (a_2/3a_1) \{ -1 \pm 2(1 - \nu - 3\nu^2) / (1 + \nu)(1 - 2\nu) \} (\exp i \alpha z) / \alpha z \tag{41b}$$

where the upper and lower signs refer to the forward and backward scattering along the  $\pm z$ -axis, respectively.

## CLOSING REMARKS

The solution to the direct scattering of an elliptical inhomogeneity is obtained by employing the extended version of Eshelby's method of equivalent inclusion and a limiting concept. When the inhomogeneity becomes a flat void the solution is appropriate for a crack with very sharp tip, i.e. a mathematical crack. The special case of longwave limit is given as an example. Since the solution given is analytic, computer display of results for different material systems and aspect ratios can easily be obtained, if needed.

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1. Report No. NASA CR-3705		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle  SCATTER OF ELASTIC WAVES BY A THIN FLAT ELLIPTICAL INHOMOGENEITY				5. Report Date July 1983	
				6. Performing Organization Code	
7. Author(s)  L. S. Fu				8. Performing Organization Report No.  None	
				10. Work Unit No.	
9. Performing Organization Name and Address  The Ohio State University Dept. of Engineering Mechanics Columbus, Ohio 43210				11. Contract or Grant No.  NAG-3340	
				13. Type of Report and Period Covered  Contractor Report	
12. Sponsoring Agency Name and Address  National Aeronautics and Space Administration Washington, D.C. 20546				14. Sponsoring Agency Code  505-53-12 (E-1654)	
15. Supplementary Notes  Final report. Project Manager, Alex Vary, Materials Division, NASA Lewis Research Center, Cleveland, Ohio 44135.					
16. Abstract  Elastodynamic fields of a single, flat, elliptical inhomogeneity embedded in an infinite elastic medium subjected to plane time-harmonic waves are studied in detail by employing the extended version of Eshelby's method of equivalent inclusion developed and given in an earlier paper. Scattered displacement amplitudes and stress intensities are obtained in series form for an incident wave in an arbitrary direction. The cases of a penny shaped crack and an elliptical crack are given as examples. The analysis is valid for $\alpha a$ up to about two, where $\alpha$ is longitudinal wave-number and $a$ is a typical geometric parameter.					
17. Key Words (Suggested by Author(s))  Nondestructive testing; Ultrasonics; Elastic waves; Fracture; Cracks; Material inhomogeneities			18. Distribution Statement  Unclassified - unlimited STAR Category 38		
19. Security Classif. (of this report)  Unclassified	20. Security Classif. (of this page)  Unclassified		21. No. of pages  17	22. Price*  A02	